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BOOK REVIEW

High-Order Methods for Incompressible Fluid Flow. By M. O. Deville, P. F. Fischer & E. H. Mund. Cambridge University Press, 2002. 528 pp. ISBN 0521453097. £60.

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Although the title might suggest coverage of a broader scope of different high-order numerical techniques, in fact this book primarily addresses the theory and implementation of spectral and spectral element methods for incompressible flows. As evidenced by many papers published in the *Journal of Fluid Mechanics*, use of spectral element methods in scientific computing of complex geometry flows has increased markedly over the past few decades. The method combines the geometric flexibility of finite element or volume methods with the fast convergence properties of spectral methods for smooth solutions. In a similar way to finite element/volume methods the solution domain is decomposed into smaller elemental regions of a specific shape which gives the approach geometric flexibility. Unlike a standard finite element/volume method, however, a high-order polynomial expansion is then applied within each element in a way reminiscent of classical spectral method. The use of nodal Lagrange polynomials as expansion bases is typically associated with spectral element methods as originally proposed by A. T. Patera (MIT). This approach has mainly been applied in the fluid mechanics field. Readers may also be familiar with a similar technique which normally uses hierarchical expansion bases and has primarily been developed for use in structural mechanics, known as the '*p*-type' finite element method.

Despite the growing interest in spectral element methods the practicalities of implementing these techniques can be quite opaque to anyone trying to develop a code outside established research groups. Early texts on the topic tended to be of a more mathematical nature although more recent books have addressed some of the practicalities of implementation. The extensive experience of the authors of this text in both the development and application of spectral element methods has resulted in the book striking a good balance between these two extremes. The presentation is developed from a reasonably mathematical point of view but the text also includes useful practical details such as parallel implementation and preconditioned iterative solution techniques. The cover states that "computer scientists, engineers, and applied mathematicians interested in developing software for solving problems will find this book a valuable reference" and this is certainly true. Relative novices in the field will, however, require a few other references on basic finite element methods and numerical quadrature.

The 500-page book starts by outlining the governing equations and explaining the need for high-order methods. The next three chapters then discuss the implementation of the technique. Starting with a one-dimensional elliptic problem, different approximation methods are outlined, with an initial discussion on iterative solution techniques in chapter 2. Temporal discretization and the treatment of parabolic and hyperbolic problems as well as the stabilization of these discretizations are then addressed in the next chapter. The extension of the technique to multiple dimensions for all mathematical types of equations is dealt with in chapter 4. At this point the reader is armed with the appropriate knowledge to tackle the fluid mechanics problems which are split over the next two chapters: the first covers the steady, and the second the unsteady, Stokes and Navier–Stokes equations. Although by this point the book would have achieved its objective of addressing "high order methods for incompressible fluid flow" there are still two very useful chapters to follow. These consider some numerical and implementational issues which must be addressed for any spectral element implementation to be of practical use in problems of scientific or engineering interest. The first deals with domain decomposition techniques such as preconditioning iterative solvers and 'mortar methods' for patching together non-conforming discretizations. The final chapter discusses vector and parallel implementations of the algorithms.

The book is clearly written, with a good distribution of helpful figures and some pseudo-code to help unravel certain aspects of the algorithms. More pseudo-code might have been helpful but I believe would have detracted from the flow of the text. Although spectral element and p-type finite elements are very similar in many ways the implementation details can be quite different. This book only handles nodal spectral element methods in detail. Although this is not a significant limitation, from a personal perspective I would have liked to have seen more of the use of p-type finite elements. It would also have been interesting to see an extension of the discussion beyond incompressible flow but as the title makes eminently clear this is not the primary focus of the text. Overall this book makes a strong contribution which enhances the available literature on this topic and endeavours to make spectral element methods more accessible to the general practitioner.

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